Math 3070/6070 Homework 5
Due: Nov 13th, 2023

1. (3.15) In class, we showed that the $Poisson(\lambda)$ distribution is the limit of the negative binomial(r, p) distribution as $r \to \infty$, $p \to 1$, $r(1-p) \to \lambda$. Show that under these conditions the mgf of the negative binomial converges to that of the Poisson.

2. (3.23) The *Pareto distribution*, with parameters α and β, has pdf

$$f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \alpha < x < \infty, \alpha > 0, \beta > 0.$$

   1. Verify that $f(x)$ is a pdf.
   2. Derive the mean and variance of this distribution.
   3. Prove that the variance does not exist if $\beta \leq 2$.

3. (3.28) Show that each of the following families is an exponential family

   1. normal family with either parameter μ or σ known.
   2. gamma family with either parameter a or b known or both unknown.
   3. beta family with either parameter a or b known or both unknown.
   4. Poisson family
   5. negative binomial family with r known, $0 < p < 1$.

4. (3.33) For each of the following families:

   1. Verify that it is an exponential family.
   2. Describe the curve on which the $\boldsymbol{\theta}$ parameter vector lies.
   3. Sketch a graph of the curved parameter space.

   (a) $n(\theta, \theta)$

   (b) $n(\theta, a\theta^2)$, a known

5. (3.37) Show that if $f(x)$ is a pdf, symmetric about 0, then μ is the median of the location-scale pdf

$$(1/\sigma)f\left((x-\mu)/\sigma\right), -\infty < x < \infty.$$