Math 3070/6070 Homework 1
Due: Sept 4th, 2023
Submit to your TA at Recitations

1. (1.1) For each of the following experiments, describe the sample space.
    1. Toss a coin four times.
    2. Count the number of insect-damaged leaves on a plant.
    3. Measure the lifetime (in hours) of a particular brand of light bulb.
    4. Record the weights of 10-day-old rats.
    5. Observe the proportion of defectives in a shipment of electronic components.
2. (1.5) Approximately one-third of all human twins are identical (one-egg) and two-thirds are fraternal (two-egg) twins. Identical twins are necessarily the same sex, with male and female being equally likely. Among fraternal twins, approximately one-fourth are both female, one-fourth are both male, and half are one male and one female. Finally, among all U.S. births, approximately 1 in 90 is a twin birth. Define the following events:

$$A = \{\text{a U.S. birth results in twin females}\}$$
$$B = \{\text{a U.S. birth results in identical twins}\}$$
$$C = \{\text{a U.S. birth results in twins}\}$$

    1. State, in words, the event $A \cap B \cap C$.
    2. Find $\Pr(A \cap B \cap C)$.
3. (1.6) Two pennies, one with $\Pr(\text{head}) = u$ and one with $\Pr(\text{head}) = w$, are to be tossed together independently. Define

$$p_0 = \Pr(0 \text{ heads occur}),$$
$$p_1 = \Pr(1 \text{ head occurs}),$$
$$p_2 = \Pr(2 \text{ heads occur}).$$

    Can u and w be chosen such that $p_0 = p_1 = p_2$? Prove your answer.
4. (1.13) If $\Pr(A) = \frac{1}{3}$ and $\Pr(B^c) = \frac{1}{4}$, can A and B be disjoint? Explain.
5. (1.27) Verify the following identities for $n \geq 2$.
    1. $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$
    2. $\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}$
    3. $\sum_{k=1}^{n} (-1)^{k+1} k \binom{n}{k} = 0$